Estimating Maximum Aircraft Icing Environments Using a Large Data Base of In-Situ Observations
AIAA 2006-0266

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Environnement Canada

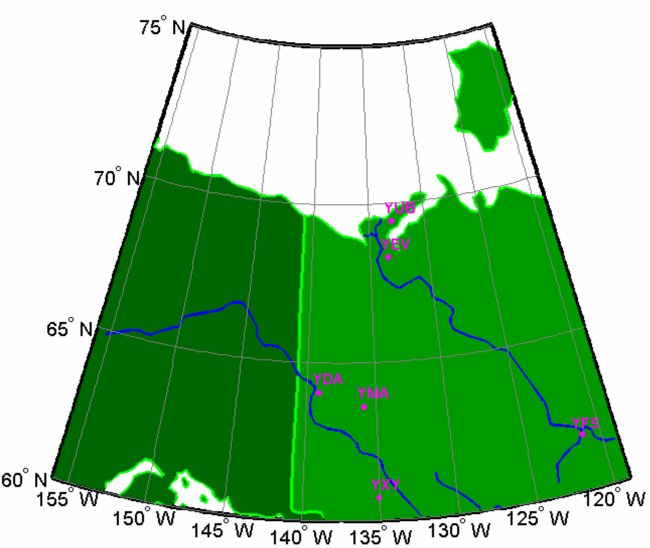
Outline

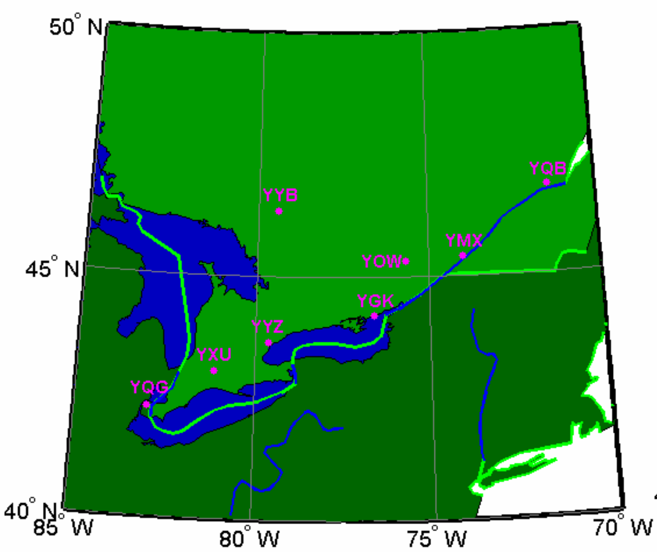
- Overview of the Data
- Extreme Value Analysis
- LWC Analysis for Environments with drops < 100 μm
- LWC Analysis for SLD Environments
- Analysis of Scale Factors
- Conclusions

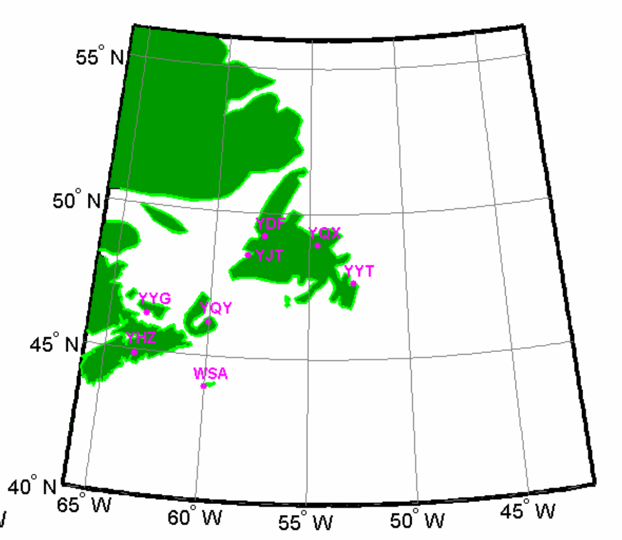
Overview of the Data

Field Projects Used For This Analysis

• CFDE I	27 Feb 1995 - 24 Mar 1995	12 Flights
• CFDE III	10 Dec1997 -18 Feb 1998	26 Flights
• FIRE.ACE	01 Apr 1998 - 29 Apr 1998	18 Flights
• AIRS I	02 Dec 1999 -19 Feb 2000	25 Flights
• AIRS I TO	02 Dec 1999 -17 Dec 1999	16 Flights
• NASA SLD	15 Jan 1997 - 27 Feb 1998	37 Flights







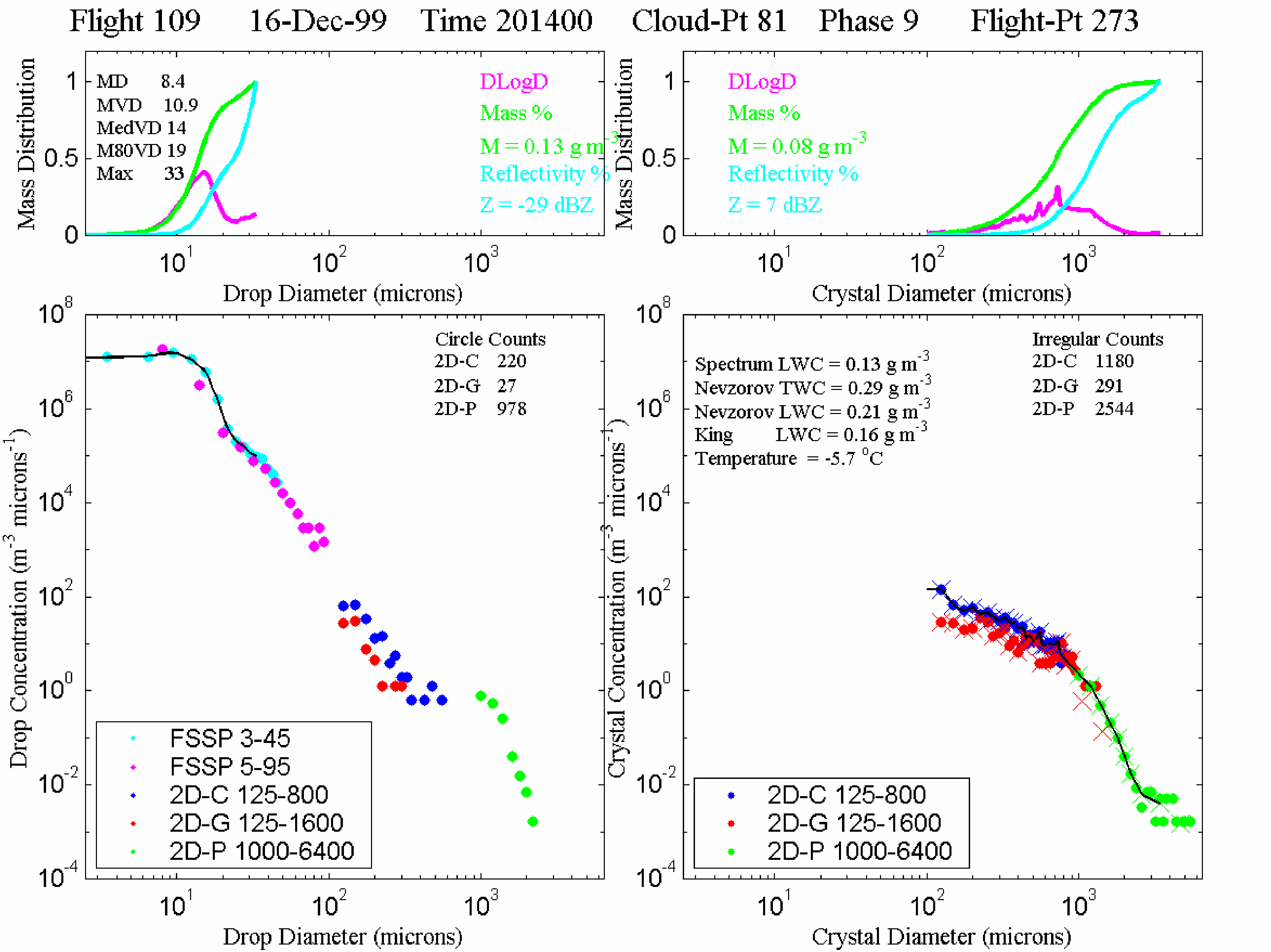
Research Aircraft

NRC

Convair-580

NASA-Glenn

Twin Otter



Flight 109 16-Dec-99 Time 201400 Cloud-Pt 81 Phase 9 Flight-Pt 273
Mass Distribution
MD 8.4
MVD 10.9
MedVD 14
M80VD 19
Max 33
DLogD
Mass %
M = 0.13 g m-3
Reflectivity %
Z = -29 dBZ
Drop Diameter (microns)
DLogD
Mass %
M = 0.08 g m-3
Reflectivity %
Z = 7 dBZ
Crystal Diameter (microns)
Drop Concentration (m-3 microns-1)
Circle Counts
2D-C 220
2D-G 27
2D-P 978
FSSP 3-45
FSSP 5-95
2D-C 125-800
2D-G 125-1600
2D-P 1000-6400
Drop Diameter (microns)
Crystal Concentration (m-3 microns-1)
Spectrum LWC = 0.13 g m-3
Nevzorov TWC = 0.29 g m-3
Nevzorov LWC = 0.21 g m-3
King LWC = 0.16 g m-3
Temperature = -5.7 °C
Irregular Counts
2D-C 1180
2D-G 291
2D-P 2544
2D-C 125-800
2D-G 125-1600
2D-P 1000-6400
Crystal Diameter (microns)

Data Point Summary

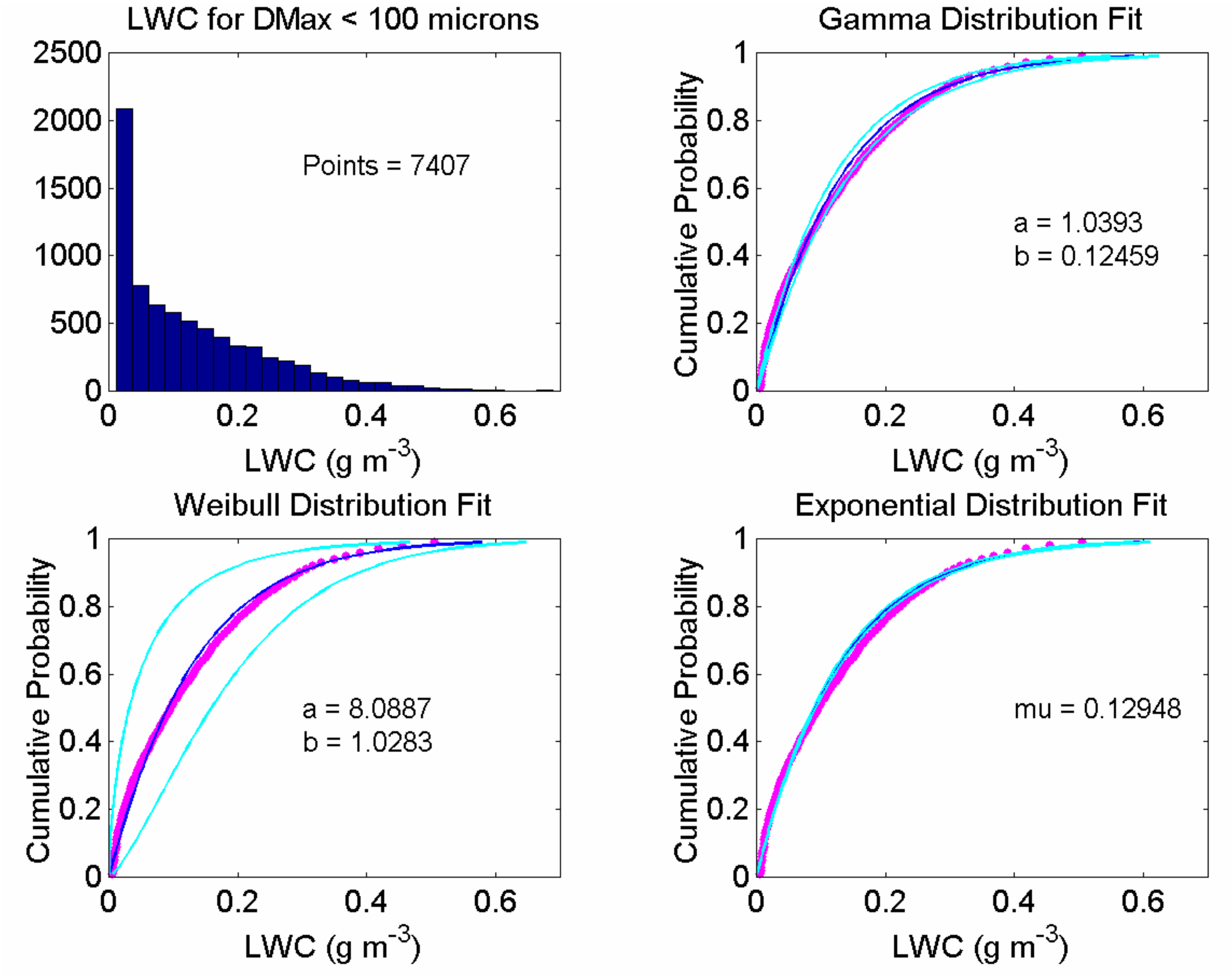
Definition	30-s Points	Characteristics
In-Flight	48301	3 km Observations
In-Cloud	27497 (57%)	TWC > 0.005 g m^{-3}
In-Cloud Cold	22263 (46%)	+ Ta ≤ 0^{o}C
In-Icing	14199 (29%)	+ L/M Phase
In-Spectra	9808 (20%)	+ Ice ≤1 L^{-1}
In-SLD	2444 (5%)	+ Dmax ≥ 100 μm

Subsets of the Data

Environment	MVD	Dmax	Points
Small Drops	< 40 µm	< 100 µm	7364
ZLE	< 40 µm	100-500 µm	1469
ZLE	> 40 µm	100-500 µm	335
ZRE	< 40 µm	> 500 µm	193
ZRE	> 40 µm	> 500 µm	447

Extreme Value Analysis

Fitting a LWC Distribution



Extreme Value Analysis

- Three types theorem: there are only three types of distribution which can arise as limiting distributions of extremes in random samples.

- Generalized Extreme Value Distribution

$$P = \exp[-(1 + \xi(\frac{x - \mu}{\psi})^{-1/\xi}]$$

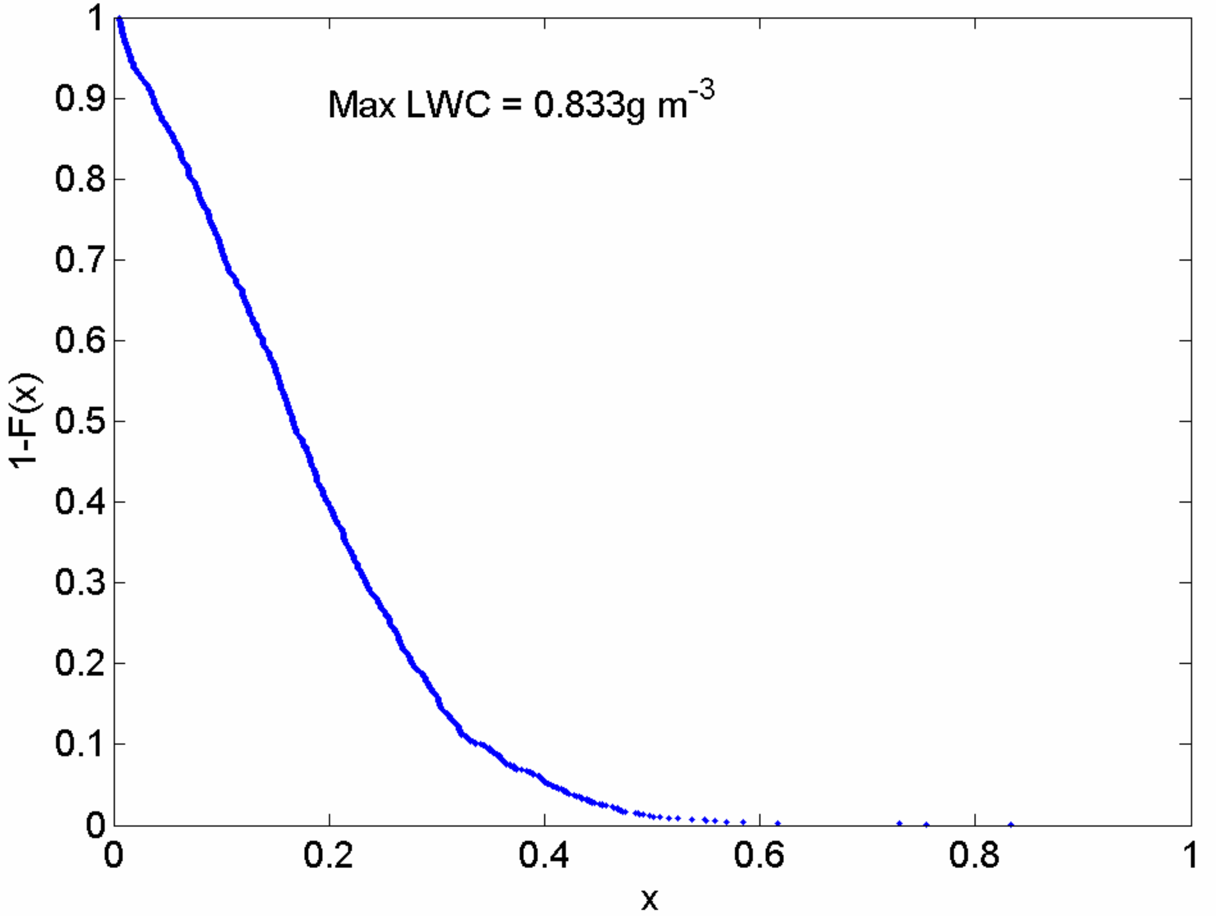
μ = location parameter

Ψ = scale parameter

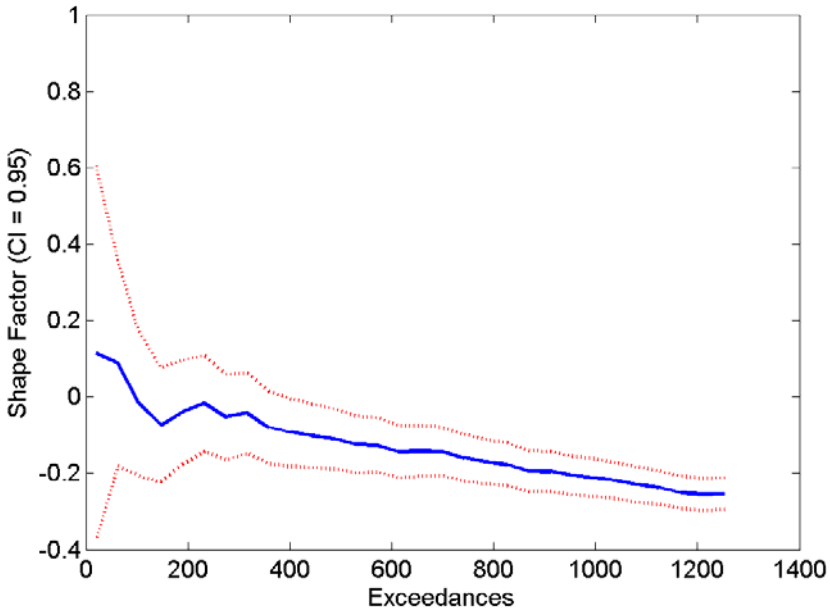
ξ = shape parameter

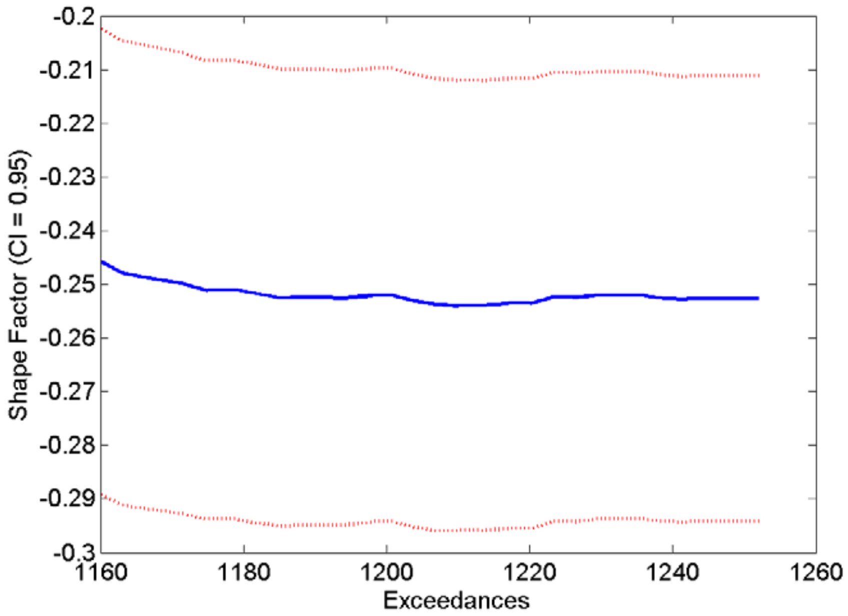
- $\xi > 0$ Fréchet family $P \sim \exp[-x^{-\alpha}]$ $\alpha = 1/\xi$ (long tail)
- $\xi < 0$ Weibull family $P \sim \exp[-(-x^{\alpha})]$ $\alpha = -1/\xi$ (short tail)
- $\xi = 0$ Gumbel family $P \sim \exp[-e^{-x}]$ (medium tail)

Empirical Distribution of LWC

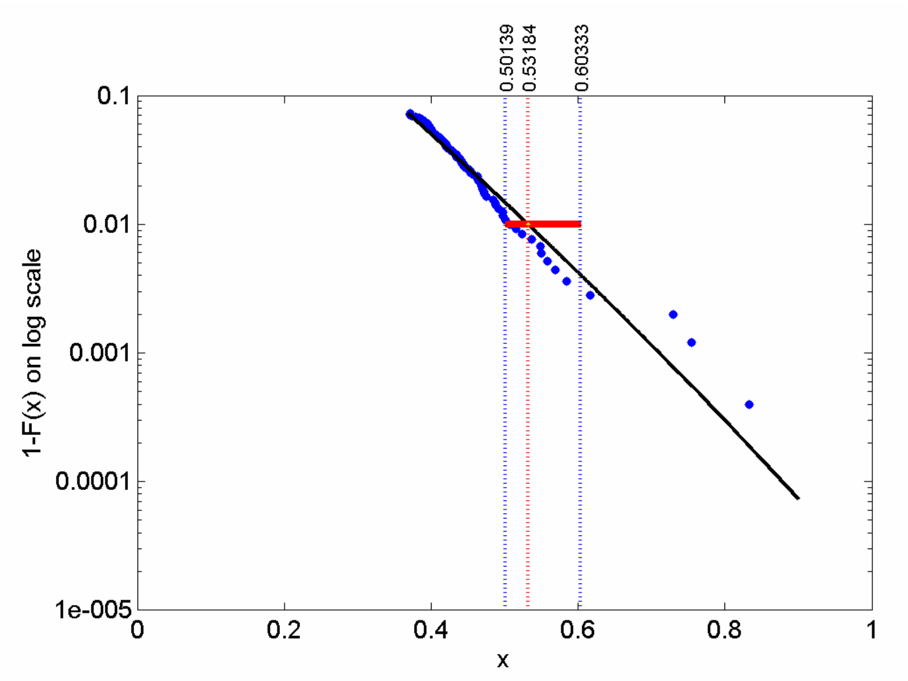


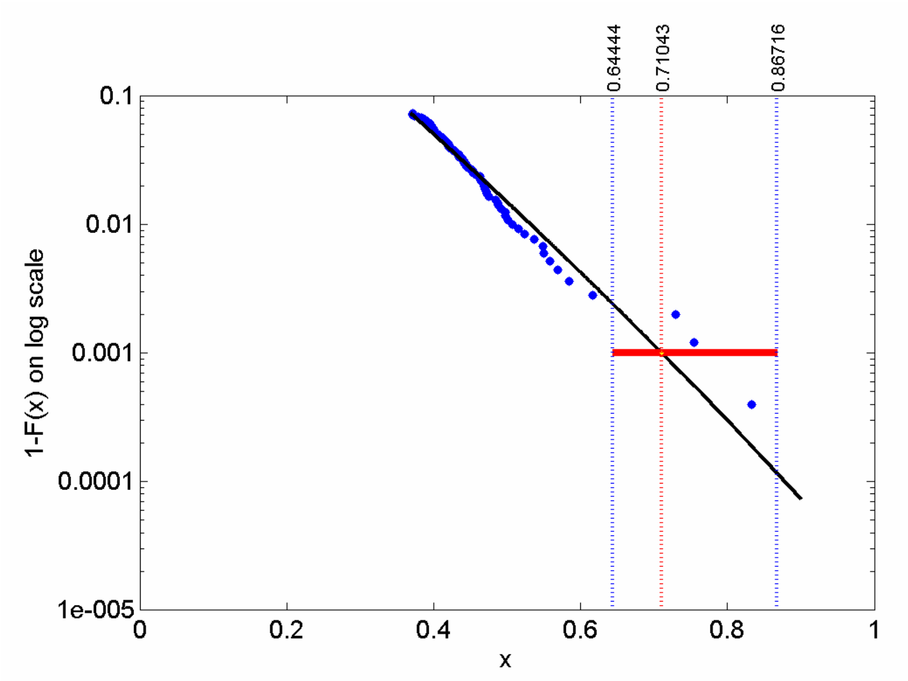
Identify Threshold for Tail





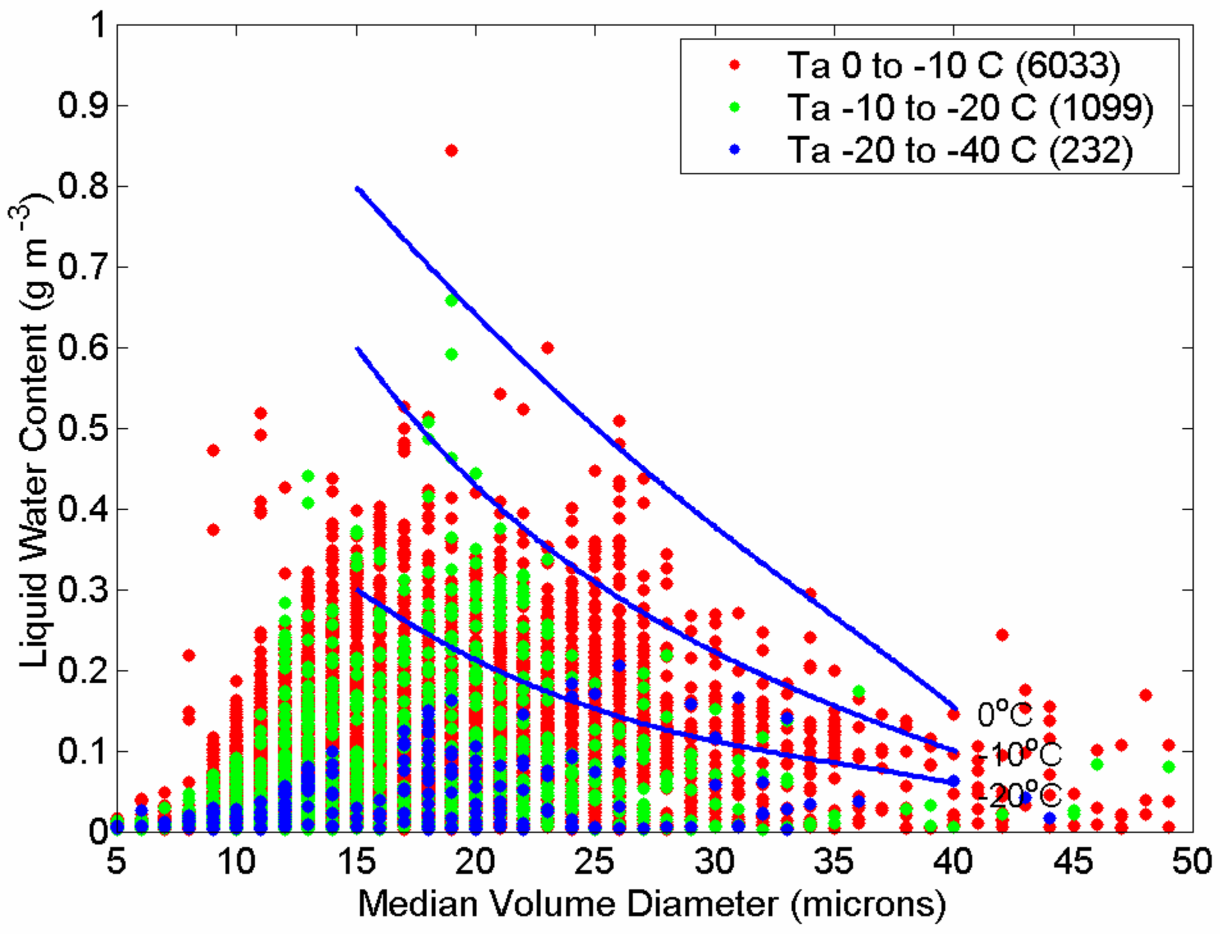
Fit the Tail to a Generalized Pareto Distribution



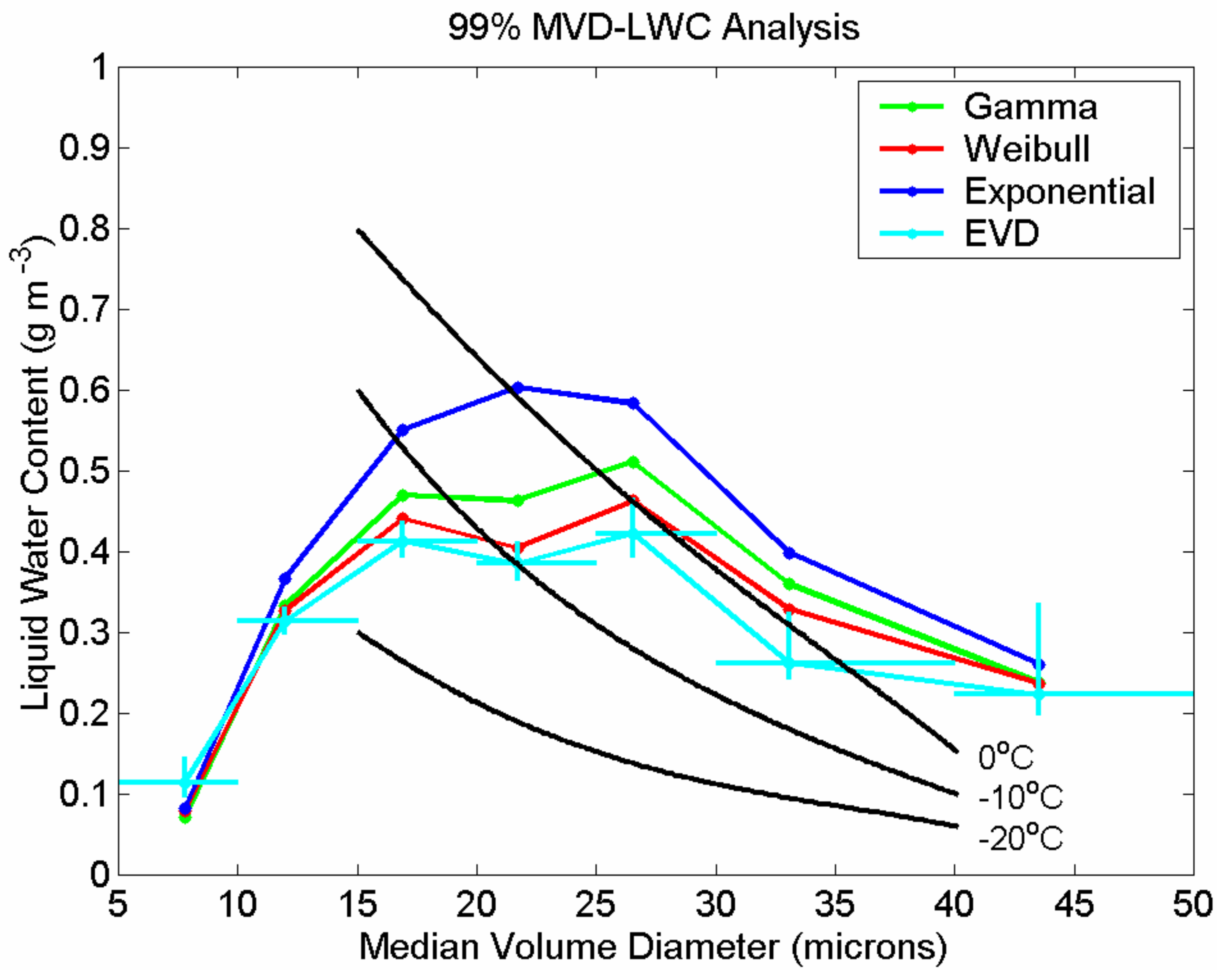


LWC Analysis for Environments With Drops < 100 µm

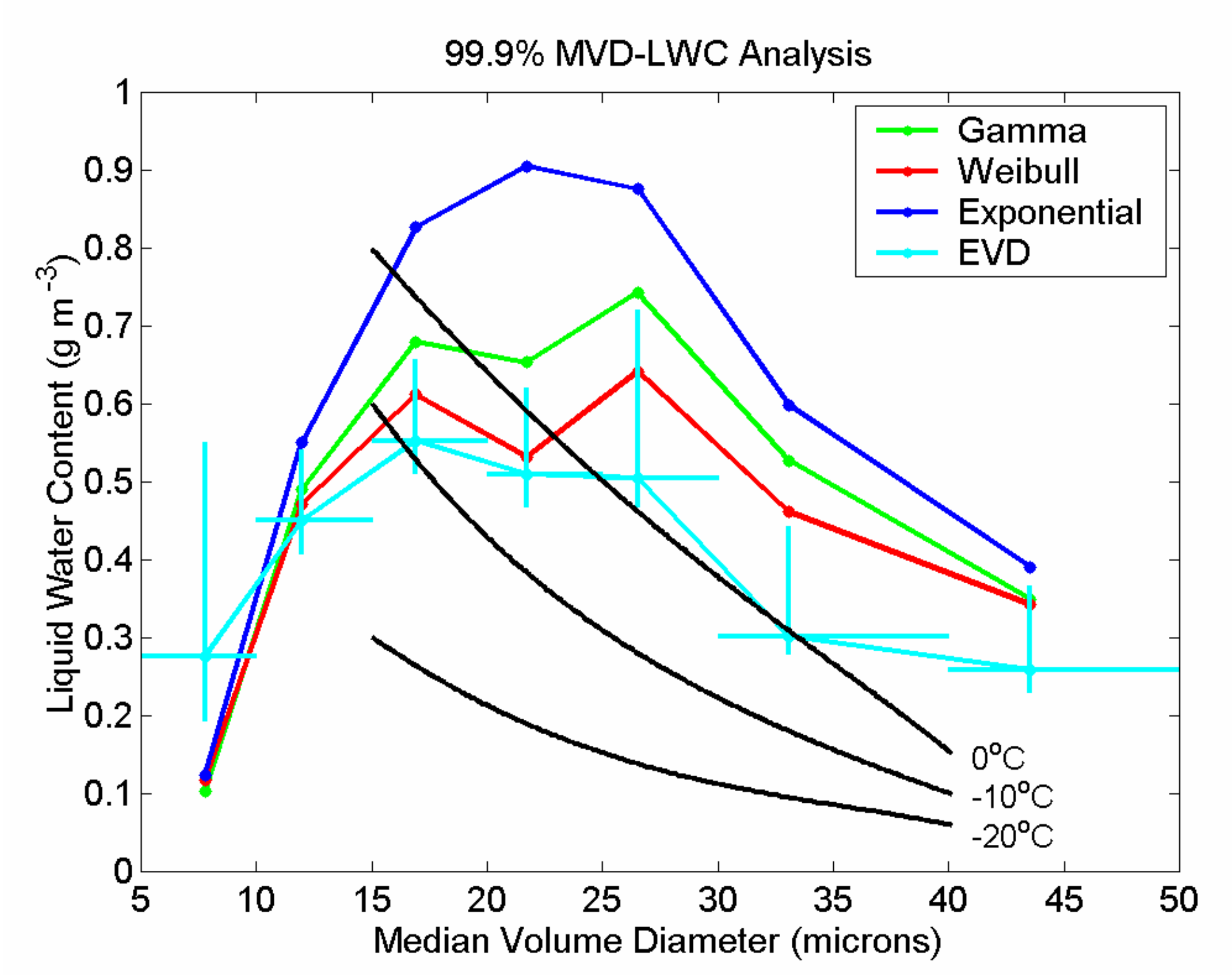
Comparison With FAR 25-C



99% LWC Environments

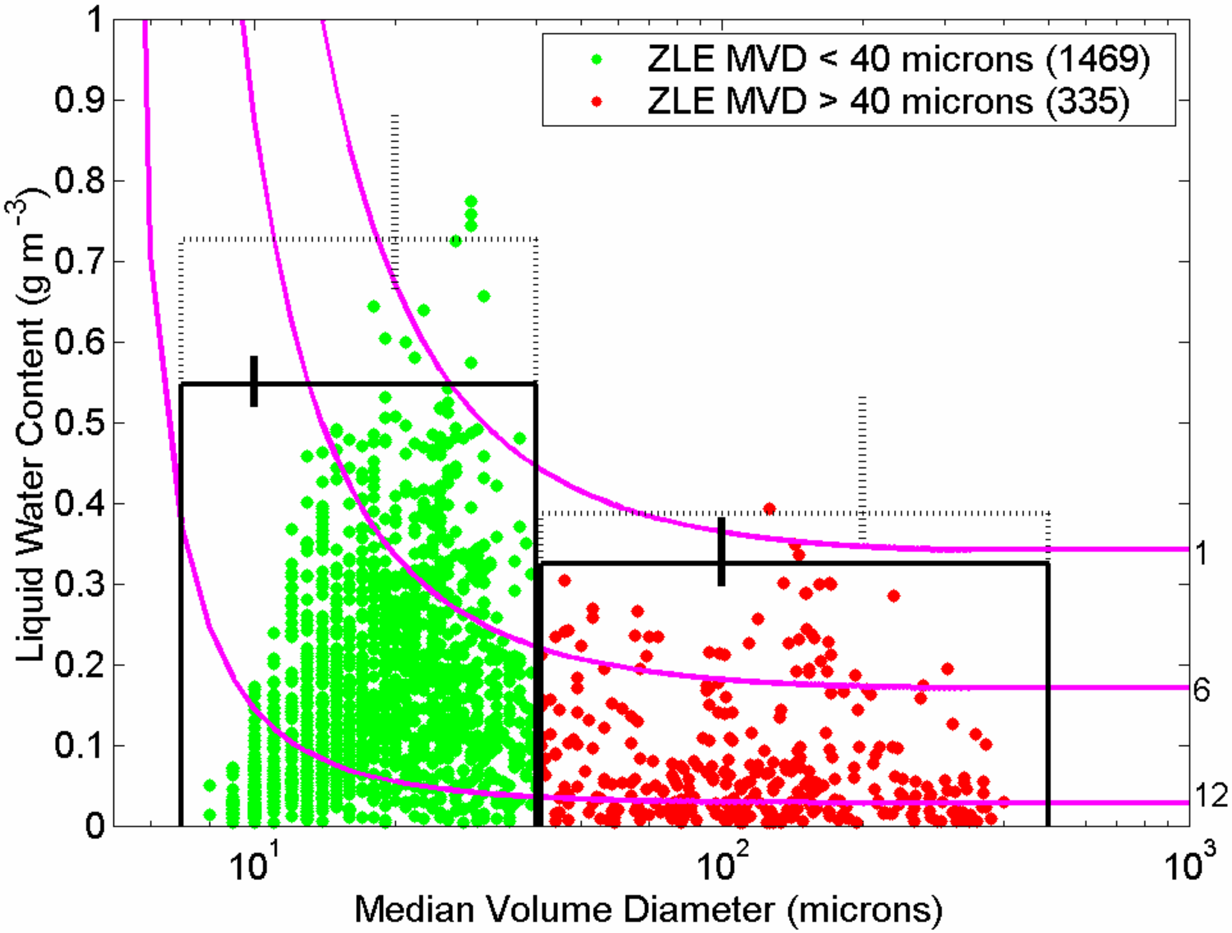


99.9% LWC Environments

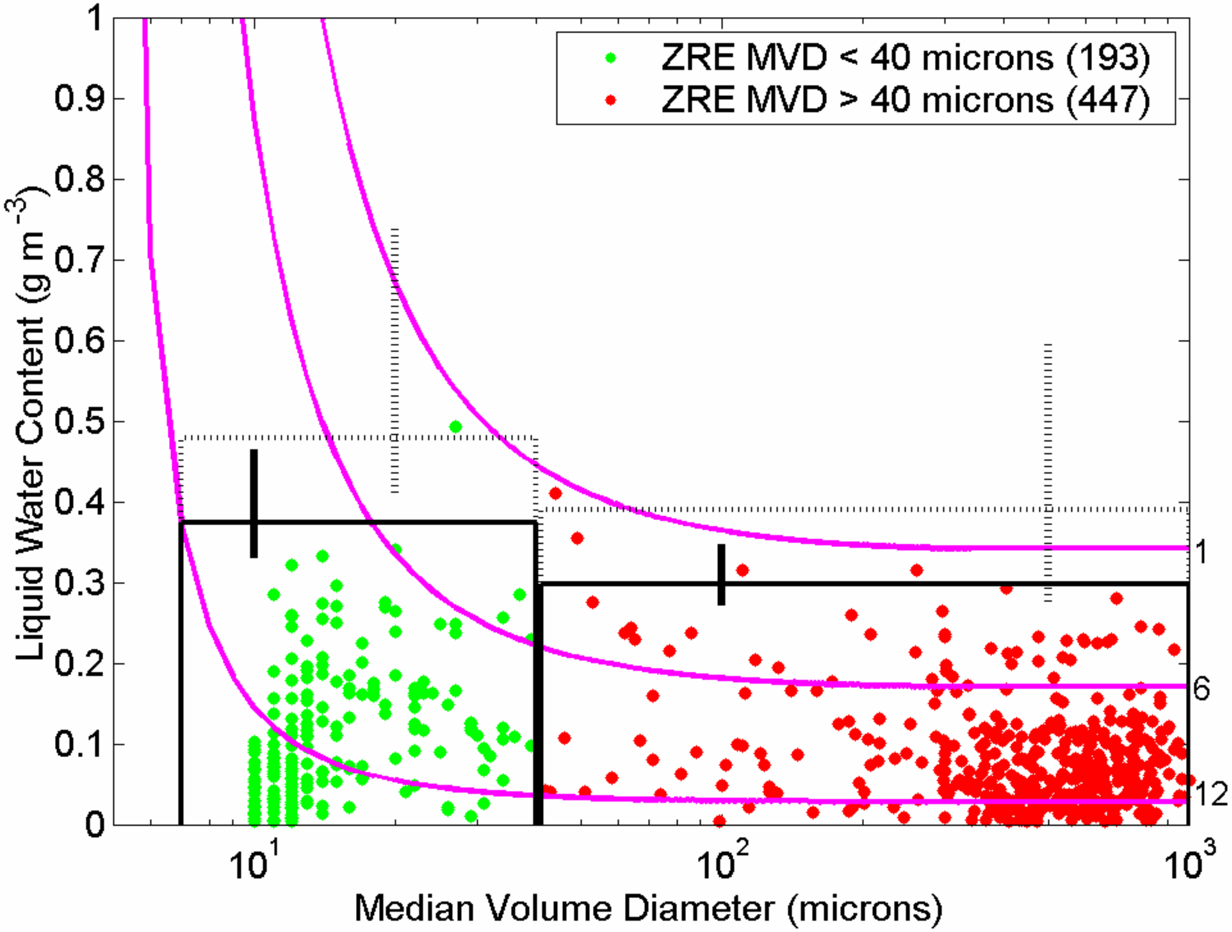


LWC Analysis for SLD Environments

LWC Analysis for SLD Environments with ZL

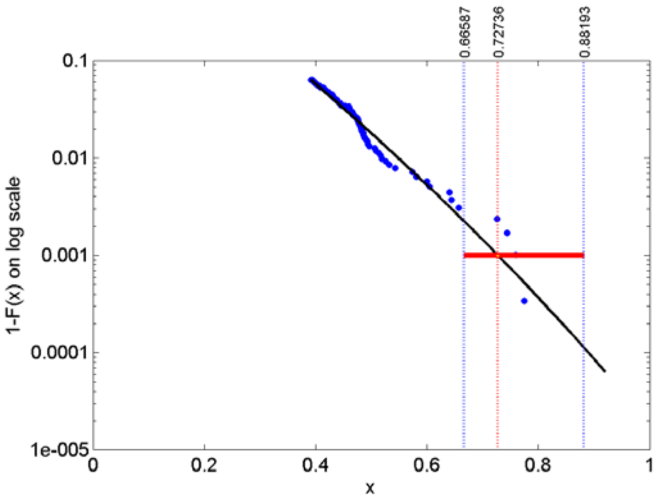


LWC Analysis for SLD Environments with ZR

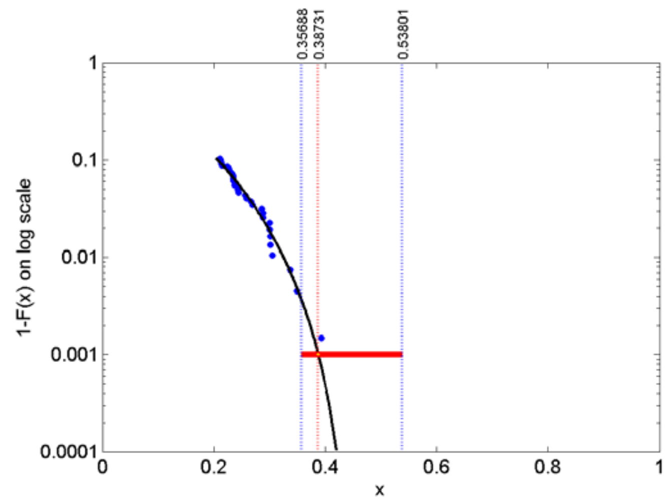


SLD Distribution Tail Fits

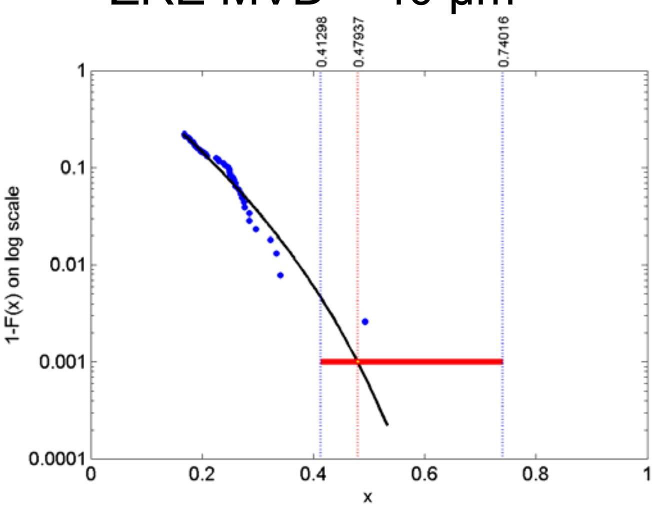
ZLE MVD < 40 µm



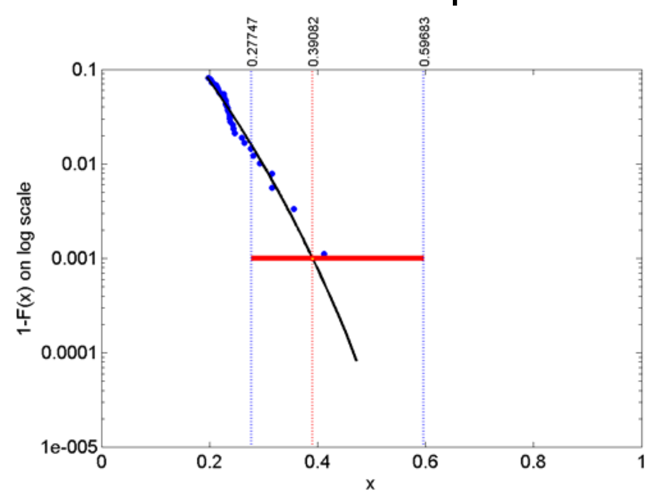
ZLE MVD > 40 µm



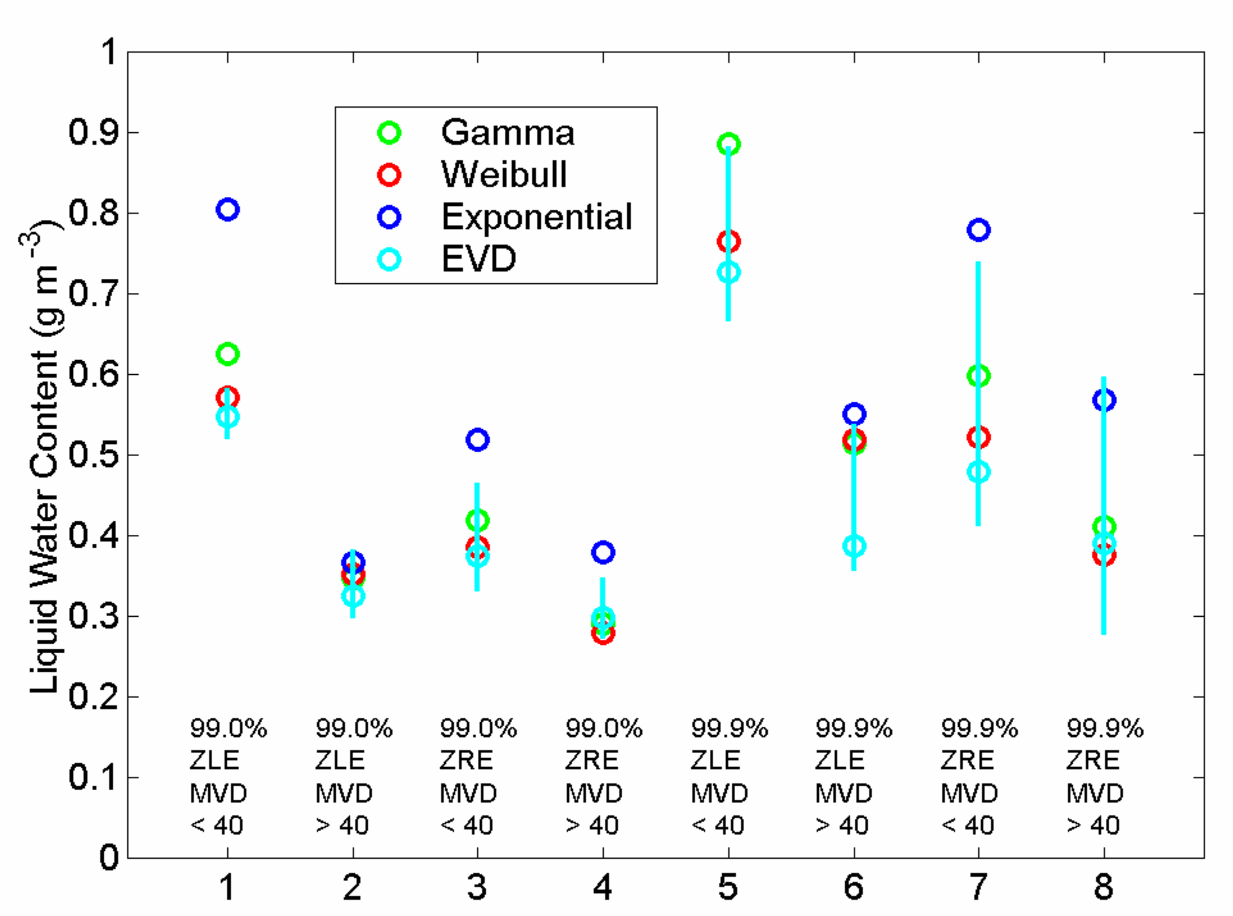
ZRE MVD < 40 µm



ZRE MVD > 40 µm

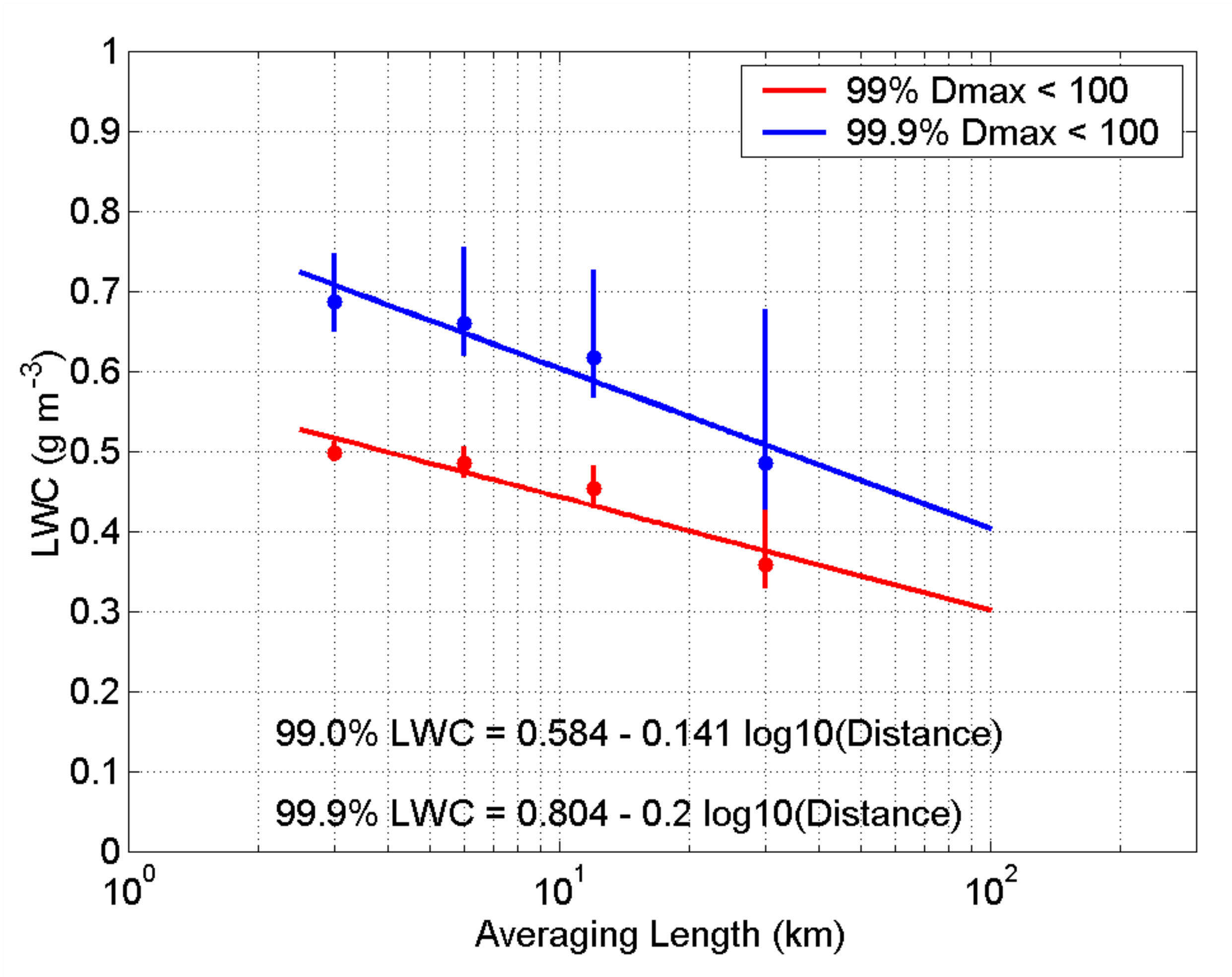


Fit Comparison for SLD LWC

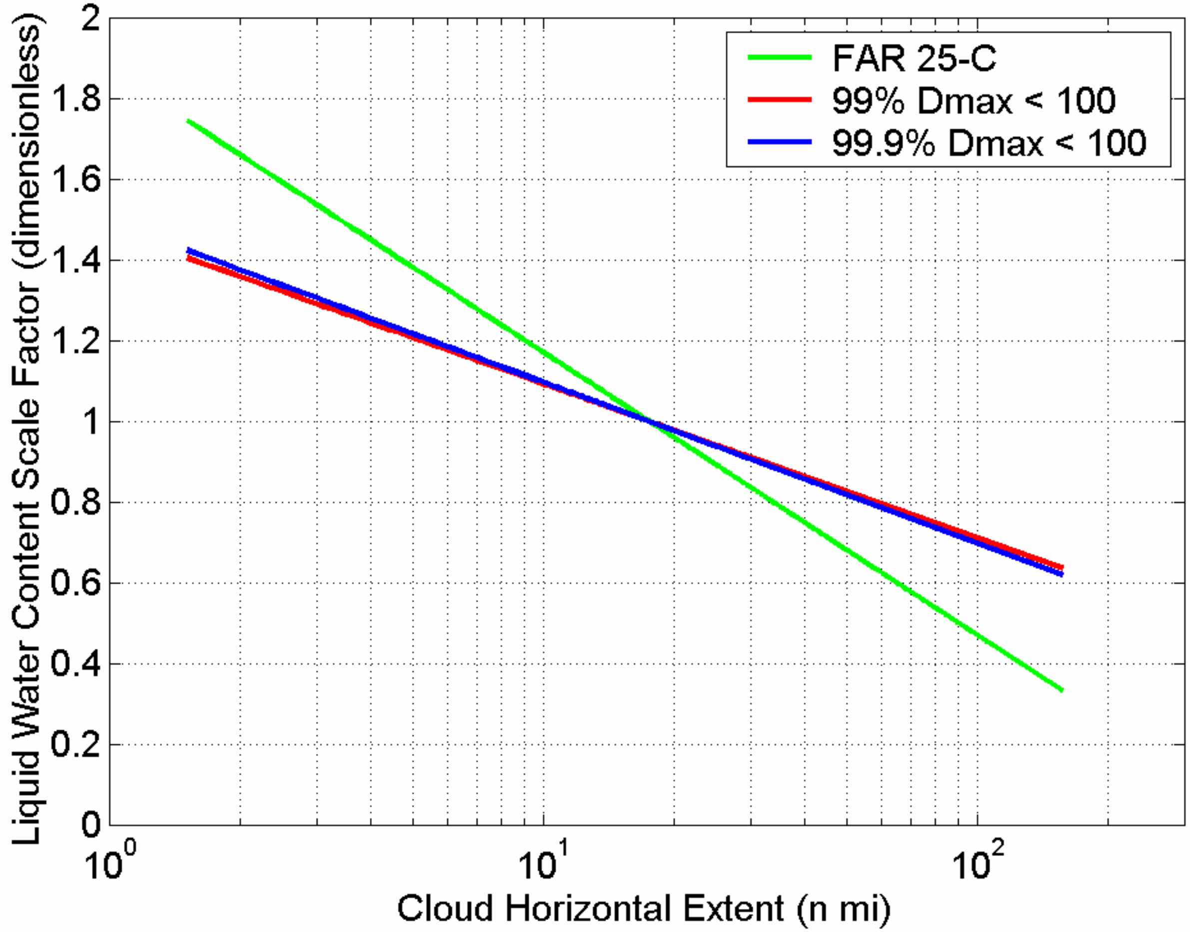


Scale Factors

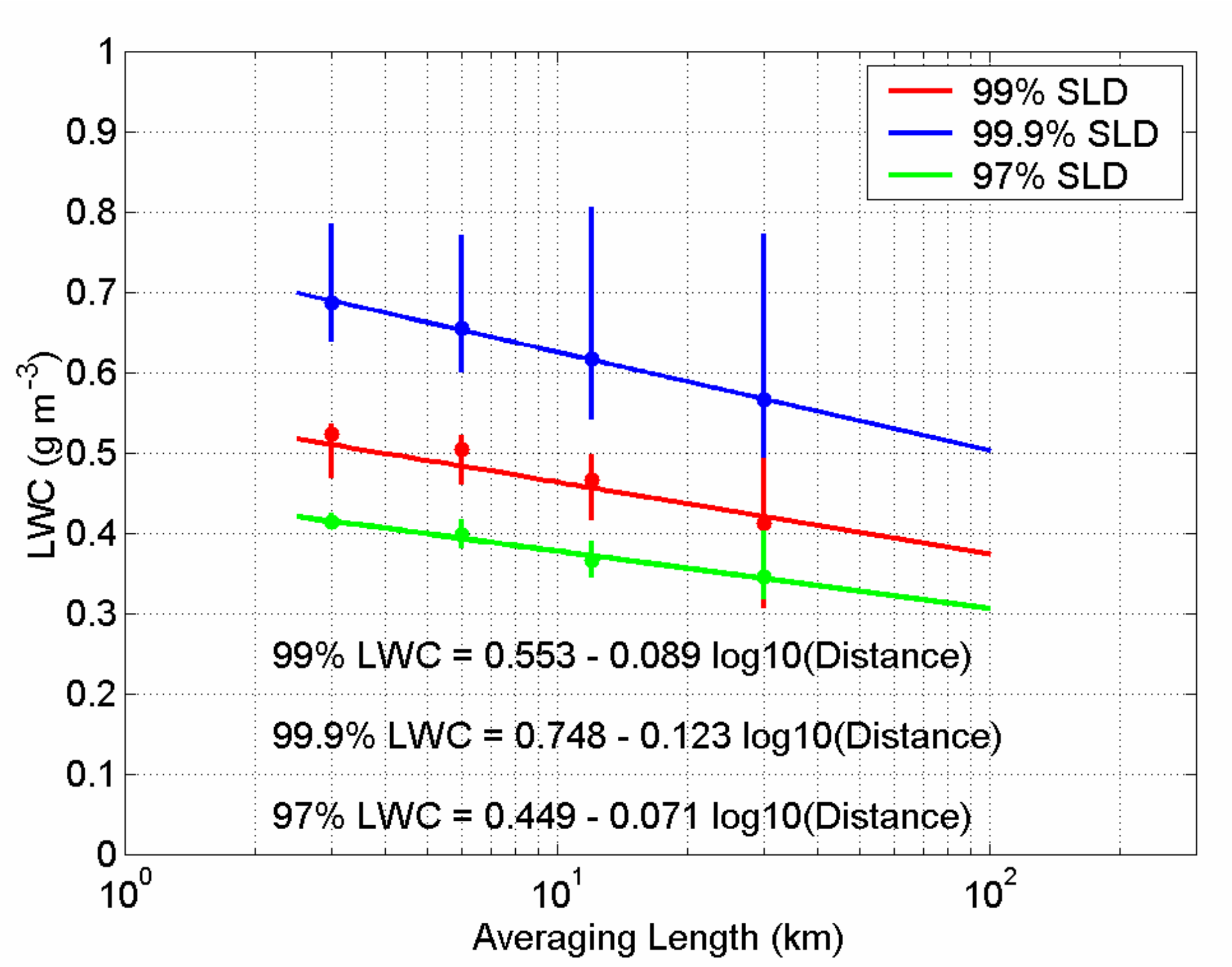
Scale Factor for Small Drop Environments



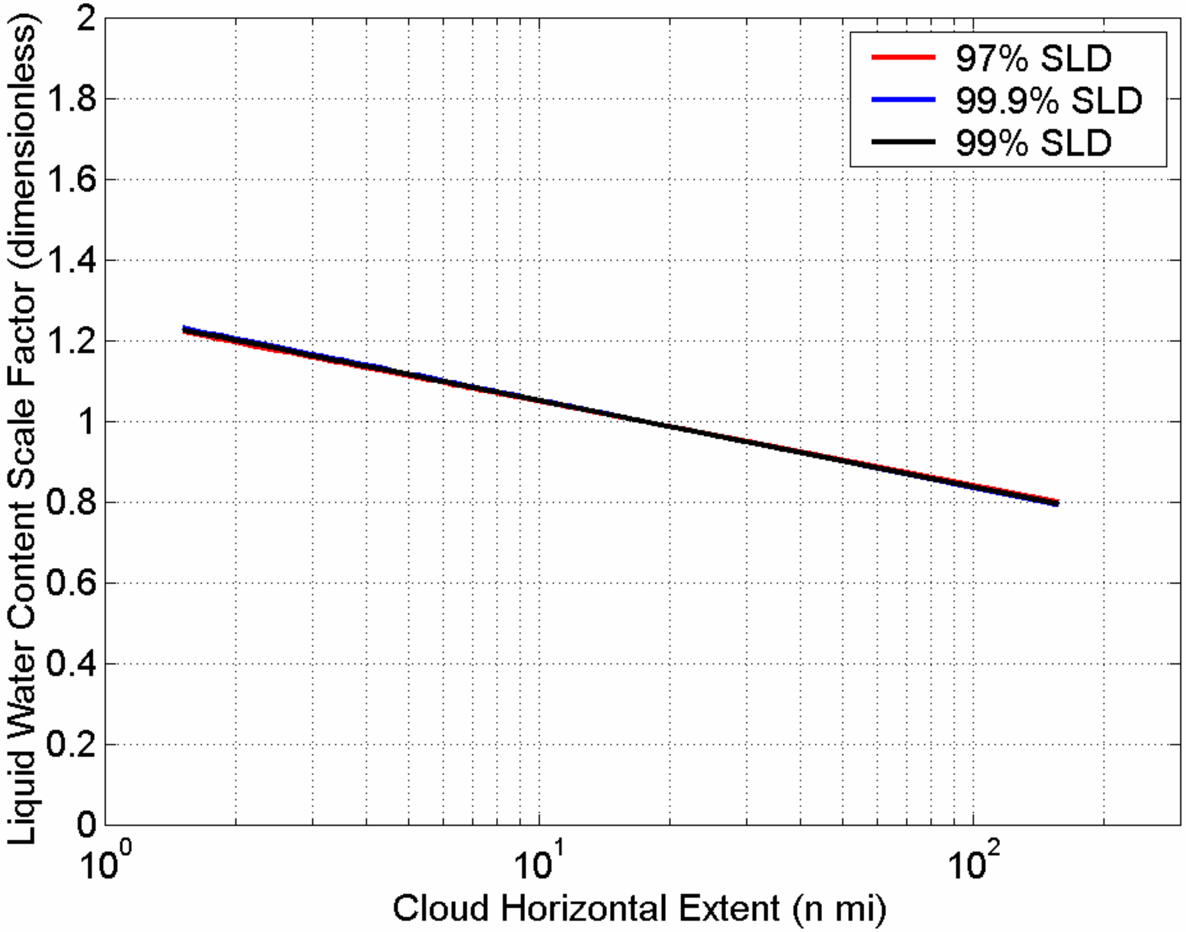
Dimensionless Scale Factor Comparison



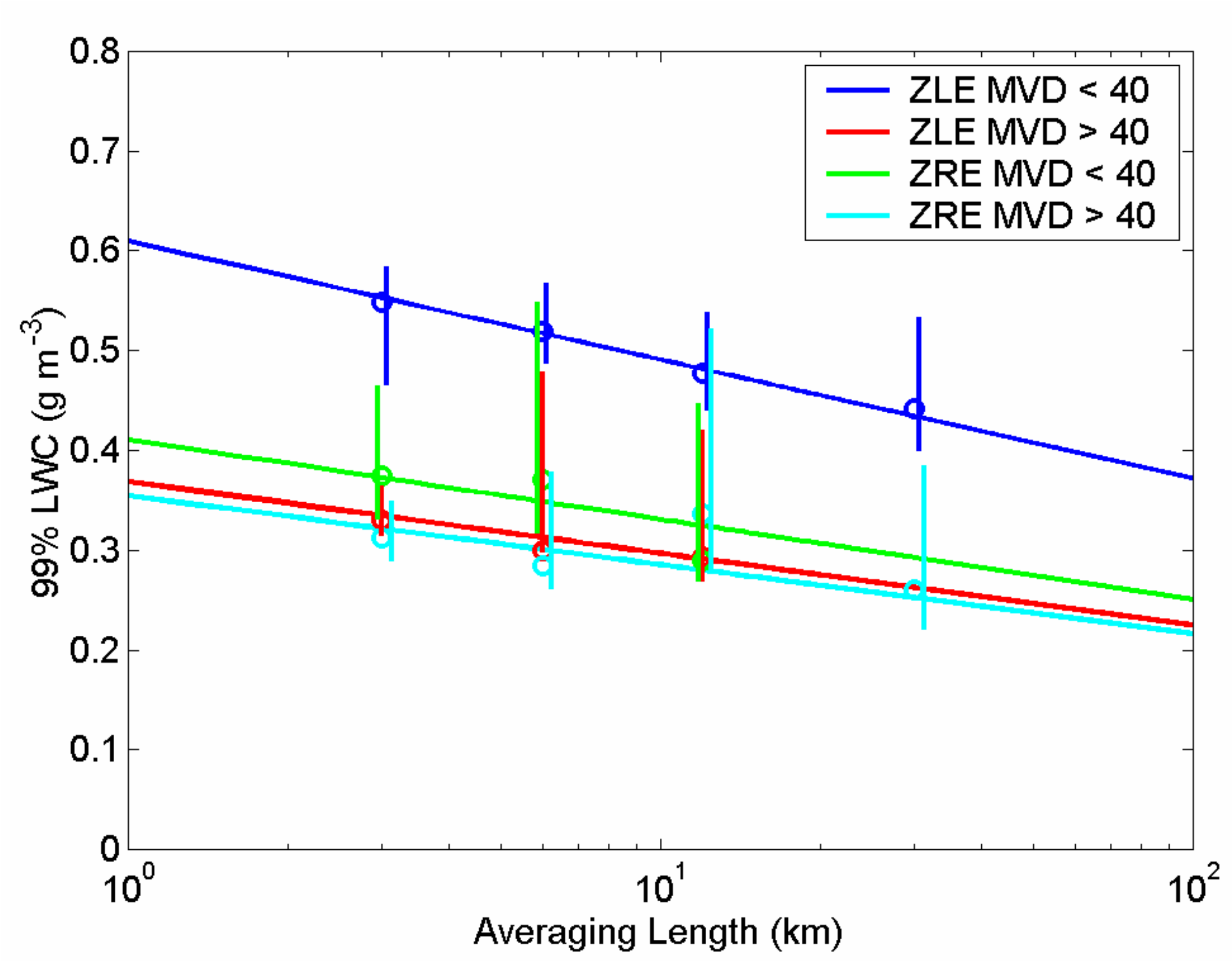
Scale Factors for SLD Environments



Dimensionless Scale Factor Comparison



Scale Factor for Each SLD Environment



Conclusions

1. For icing conditions with drops < 100 μm in diameter, the 99.9% LWC values derived using extreme value analysis techniques are consistent with the FAR 25-C 99.9% envelopes for continuous maximum icing conditions for MVD values in the range 20-35 μm.
2. When the icing conditions with drops < 100 μm in diameter are fitted to an exponential distribution, the dimensionless scale factor derived for computing horizontal extent agrees with the FAR 25-C dimensionless scale factor within 8%. However, when the data are fitted to an extreme value distribution, the scale factor has a significantly lower slope.
3. The 99% and 99.9% LWC values were determined for four subsets of icing conditions associated with SLD environments with drops > 100 μm in diameter. The extreme LWC values were computed using an extreme value distribution and compared with fits from exponential, gamma and Weibull distributions. In general, the latter fits overestimated the LWC values relative to the EVD, although the Weibull distribution fits tended to agree within the 95% confidence limits. A dimensionless scale factor for SLD conditions was derived.
4. Since extreme value statistical analysis allows quantification of the nature of distributions in the tails of the distributions, and hence provides a more accurate method for determining extreme values, it is suggested that it be used when developing future envelopes in support of aircraft certification programs.

Questions?